

Torsion: - The differentiation of \vec{b} w.r.t 's' is called torsion vector whose magnitude is denoted by τ

i.e $\tau = \left| \frac{d\vec{b}}{ds} \right|$

The reciprocal of torsion is called radius of torsion it is denoted by ρ

i.e $\rho = \frac{1}{\tau}$
 or $\boxed{\rho\tau = 1}$

Skew curvature: \rightarrow The differentiation of n w.r.t 's' is called skew curvature whose magnitude is denoted by $\sqrt{K^2 + \tau^2}$

Theorem: - A necessary and sufficient condition for a curve to be a straight line is that the curvature $K=0$ at all points of curve.

Proof: - Necessary Condition: - Let curve is straight line then

show $K=0$

The eqn of straight line in vector form

$r = as + b$ ①

where $a = \text{slope}$ and $b = \text{constant}$

differentiate ① w.r.t 's' we get

$r' = a$

but $\frac{dr}{ds} = t$

$\Rightarrow t = a$

differentiate w.r.t 's' we get

$\frac{dt}{ds} = 0$

taking mode on both side

$\left| \frac{dt}{ds} \right| = 0 \Rightarrow \boxed{K=0}$

Sufficient condition: - Let $K=0$ then show curve is the straight line